Enrollment No: _____ Exam Seat No: _____

C.U.SHAH UNIVERSITY
Summer Examination-2018

Subject Name: Transform Theory

Subject Code: 4SC05TTE1 **Branch: B.Sc. (Mathematics)**

Semester: 5 **Date: 03/04/2018** **Time:10:30 To 01:30** **Marks: 70**

Instructions:
(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Find the value of $1 * 1$, where $*$ denote convolution product. (01)

b) Find the period of$\cos mt$. (01)

c) In the Fourier series expansion of $f(x) = |x|$ in $(-\pi, \pi)$, the value of b_n is …… (01)

d) $L^{-1}\{\int_s^\infty \bar{f}(u)\, du\} =$…… . (01)

e) Find the Z-transform of unit impulse sequence $\delta(n)$. (01)

f) Find$L^{-1}\left(\frac{1}{s-a}\right)$. (01)

g) $L[f'(t)] =$ …… . (01)

h) If $F(\lambda)$ is the Fourier transform of $f(t)$, then $F[f(x-a)] =$……. (01)

i) Find the Laplace transform of $f(t) = t^{\frac{2}{3}}$. (01)

j) Evaluate the inverse Laplace transform of$\bar{f}(s) = \frac{1}{\sqrt{s}}$. (01)

k) $f(x) = e^{x^2}, -2 < x < 2$, is even function. Determine whether the statement is True or False. (01)

l) If $L[f(t)] = \bar{f}(s)$, then $L[e^{at} f(t)] = \bar{f}(s-a)$. Determine whether the statement is True or False. (01)

m) If $Z(u_n) = U(z)$, then $Z(a^{-n}u_n) = U(az)$. Determine whether the statement is True or False. (01)

n) If $F(\lambda)$ is the Fourier transform of $f(x)$, then $\mathcal{F}[f(ax); \lambda] = \frac{1}{a} F\left(\frac{\lambda}{a}\right); a > 0$. Determine whether statement is True or False. (01)

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Find the Fourier series of $f(x) = \begin{cases} x+1, -1 < x < 0 \\ x-1, \quad 0 < x < 1 \end{cases}$ with period 2. (05)

b) Find the Fourier series for the periodic function $f(x) = \begin{cases} -\pi; -\pi < x < 0 \\ x; \quad 0 < x < \pi \end{cases}$. (05)

c) Find half-range sine series for e^x in $0 < x < 1$. (04)

Q-3 **Attempt all questions** **(14)**

a) Prove that $F_s\big(x\,f(x)\big) = -\frac{d}{d\lambda}[F_c(\lambda)]$ and $F_c\big(x\,f(x)\big) = \frac{d}{d\lambda}[F_s(\lambda)]$. (05)

b) Find Fourier sine and cosine transform of $f(x) = x$. (05)

c) Show that the Fourier transform of $e^{-\frac{x^2}{2}}$ is $e^{-\frac{s^2}{2}}$. (04)

Q-4 **Attempt all questions** **(14)**

a) Evaluate $L^{-1}\left\{\frac{2s^2-6s+5}{s^3-6s^2+11s-6}\right\}$. (05)

b) Using Laplace transform, determine the solution of $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{-t}$, $y(0) = y'(0) = 0$. (05)

c) Find Laplace transform of the function $f(t) = \begin{cases} \frac{t}{a}, & 0 < t < a \\ 1, & t > a \end{cases}$. (04)

Q-5 **Attempt all questions** **(14)**

a) If $Z(u_n) = U(z)$, then prove that (05)

$$Z(u_{n+k}) = z^k\left[U(z) - u_0 - u_1 z^{-1} - u_2 z^{-2} - \cdots - u_{k-1} z^{-(k-1)}\right].$$

b) If $U(z) = \frac{2z^2+5z+14}{(z-1)^4}$, then evaluate u_2 and u_3. (05)

c) Evaluate $L^{-1}\left\{\frac{3s-2}{s^{\frac{3}{2}}} + \left(\frac{\sqrt{s}-1}{s}\right)^2 + \frac{4s-18}{9-s^2}\right\}$. (04)

Q-6 **Attempt all questions** **(14)**

a) If $f(t)$ is a periodic function with period T, then prove that (05)

$L[f(t)] = \frac{\int_0^T e^{-st} f(t)\,dt}{1-e^{-sT}}$.

b) Find the Fourier transform of the function $f(x) = \begin{cases} 1-x^2, & -1 < x < 1 \\ 0, & otherwise \end{cases}$. (05)

c) Find Laplace transform of the function $f(t) = \sin\sqrt{t}$. (04)

Q-7 **Attempt all questions** **(14)**

a) Find the Fourier series of the function $f(x) = \begin{cases} -k, & -\pi < x < 0 \\ k, & 0 < x < \pi \end{cases}$. Also deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$. (05)

b) Find the Laplace transform of $f(t) = \frac{\cos at - \cos bt}{t} + t\sin at$. (05)

c) Find the Z-transform of $3n - 4\sin\frac{n\pi}{4} + 5a$. (04)

Q-8 **Attempt all questions** **(14)**

a) Find sine and cosine integral of $f(x) = e^{-kx}$. (05)

b) Find $f(x)$ if its cosine transform is $F_c(\lambda) = \begin{cases} \frac{2a-\lambda}{4\pi}, & \lambda < 2a \\ 0, & \lambda \geq 2a \end{cases}$. (05)

c) Prove that $Z[\cos n\theta] = \frac{z(z-\cos\theta)}{z^2-2z\cos\theta+1}$ and $[\sin n\theta] = \frac{z\sin\theta}{z^2-2z\cos\theta+1}$. (04)